Rewarding Learning

General Certificate of Secondary Education

2008

Additional Mathematics

Paper 1
Pure Mathematics

[G0301]

G0301

TUESDAY 13 MAY, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet and the Supplementary Answer Booklet provided.
Answer **all eleven** questions.
At the conclusion of this examination attach the Supplementary Answer Booklet to your Answer Booklet using the treasury tag supplied.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
A copy of the formulae list is provided.

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Answer **all eleven** questions

1 **(i)** Using the axes and scales in **Fig. 1** in your Supplementary Answer Booklet, **sketch** the graph of $y = \sin x$ for $-180° \leqslant x \leqslant 180°$. [2]

(ii) Using the axes and scales in **Fig. 2** in your Supplementary Answer Booklet, **sketch** the graph of $y = \sin 2x$ for $-180° \leqslant x \leqslant 180°$. [2]

2 **(i)** Solve the equation

$$\tan \theta = 2$$

for $0° \leqslant \theta < 360°$. Give your answers correct to 2 decimal places. [2]

(ii) **Hence** solve the equation

$$\tan (4x + 5°) = 2$$

for $0° \leqslant x < 90°$. Give your answers correct to 2 decimal places. [3]

3 **(i)** Find $\mathbf{A}^{-1}$ where $\mathbf{A} = \begin{bmatrix} 4 & -3 \\ -7 & 6 \end{bmatrix}$ [2]

(ii) **Hence**, using a matrix method, solve the following simultaneous equations for x and y.

$$4x - 3y = -5$$

$$-7x + 6y = 8$$ [4]

4 **(a)** Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $y = 3x^4 + \dfrac{4}{x^5} - 1$ [2]

(b) Find $\displaystyle\int \left(3x^4 + \frac{4}{x^5} - 1\right) \mathrm{d}x$ [4]

5 **Fig. 3** shows a sketch of the graph of $y = -x^3 + 6x^2 - 5x$

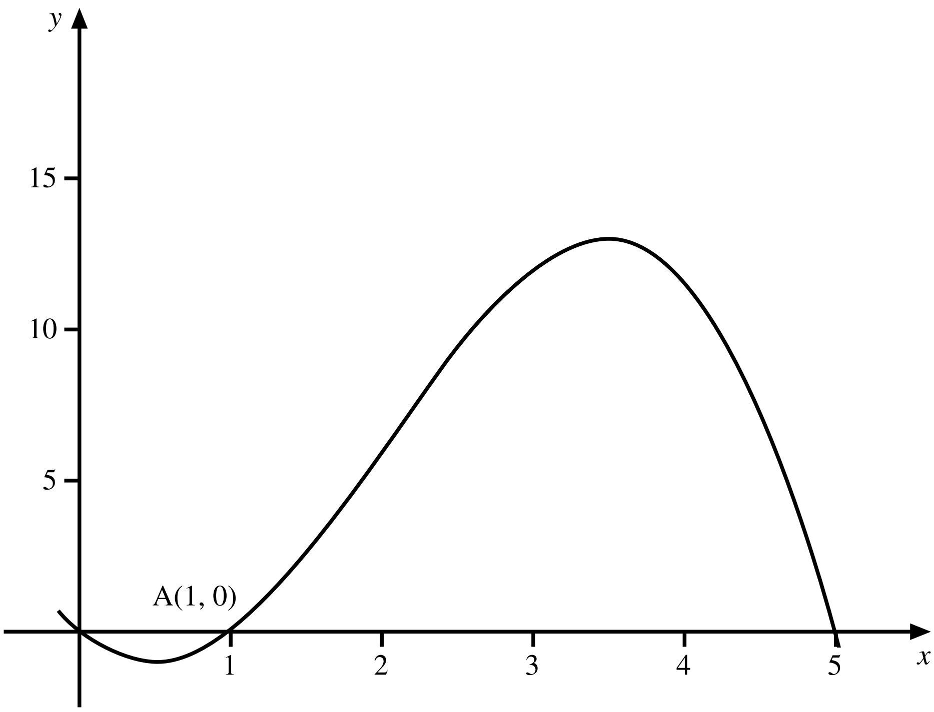

Fig. 3

(i) Find the equation of the tangent to the curve at the point A(1,0). [4]

The tangent at another point B on the curve is parallel to the tangent at A.

(ii) Find the coordinates of B. [4]

6 **(i)** Show that

$$\frac{2x-1}{3x+4} - \frac{2x+1}{4-x}$$

can be written as

$$\frac{2(4x^2+x+4)}{3x^2-8x-16}$$ [4]

(ii) **Hence**, or otherwise, solve the equation

$$\frac{2x-1}{3x+4} - \frac{2x+1}{4-x} = 2$$ [4]

7 **(a)** If $\log_a 32 = 5$ what is the value of a? [1]

(b) If $\log_2 5 = b$ express $\log_2 50$ in terms of b. Give your answer in its simplest form. [3]

(c) Solve the equation

$$3^{\left(7-\frac{x}{2}\right)} = 8$$

giving your answer correct to 2 decimal places. [4]

8 At a given instant one boat is at A and another is at B. The boat at A is heading at a constant speed in a straight line towards a harbour at Y. The boat at B is heading at the same speed in a straight line towards another harbour at X. The distances AY and BY are 22.00 km and 18.50 km respectively. The boat at B is due north of Y and the angle $A\widehat{Y}B$ is 56.45°, as shown in **Fig 4**.

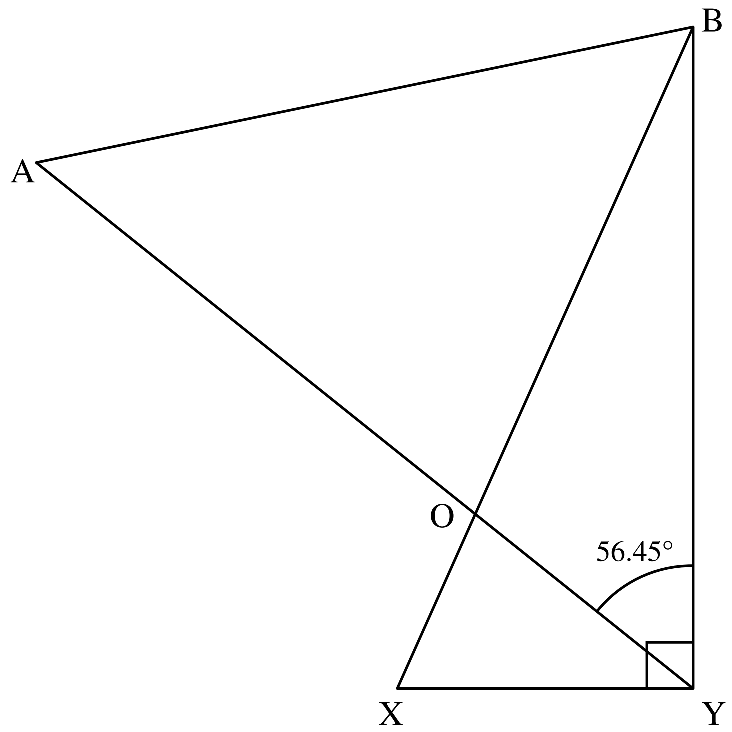

Fig. 4

(i) Show that the distance AB is 19.40 km. [3]

(ii) Calculate the size of the angle $B\widehat{A}Y$. [3]

(iii) Calculate the size of the angle $A\widehat{B}Y$. [1]

The harbour at X is 6.11 km due west of the harbour at Y.

(iv) Calculate the size of the angle $X\widehat{B}Y$. [2]

(v) Show that the angles $B\widehat{A}Y$ and $A\widehat{B}X$ are approximately equal. [1]

The lines AY and BX intersect at the point O.

(vi) Explain why the boats will meet each other at O. [1]

9 Cepheid variables are stars whose brightness fluctuates regularly with time. An astronomer measured the period P and the average luminosity (intrinisic brightness relative to the sun) L of several stars and the results are given in **Table 1**.

Table 1

Period P (days)	**Average luminosity** L (suns)
5.32	2017
11.61	4947
25.18	12 050
38.49	19 630
51.42	27 390

It is believed that a relationship of the form

$$L = kP^n$$

exists between L and P, where k and n are constants.

(i) Using **Fig. 5** in your Supplementary Answer Booklet verify this relationship by drawing a suitable straight line graph, using values correct to two decimal places.
Label the axes clearly. [6]

(ii) Hence, or otherwise, obtain values for k and n. [4]

(iii) Use the formula $L = kP^n$ with the values you obtained for k and n to calculate the average luminosity of the star X-Cygni, which has a period of 16.39 days. [1]

(iv) Use the formula $L = kP^n$ to calculate the period of the star Y-Cygni, whose average luminosity is 1394 suns. State any assumption about the formula which you make. [3]

10 Becky, Catriona and Grace went shopping in a continental market. They each bought identical sunglasses, bag and watch.

By bargaining with the stall holders Becky bought her sunglasses for 60% of the marked price, her bag for 40% of the marked price and her watch for 40% of the marked price. She spent a total of 40 euro.

Let x, y and z represent the marked prices, in euro, of the sunglasses, the bag and the watch respectively.

(i) Show that x, y and z satisfy the equation

$$3x + 2y + 2z = 200$$ [2]

Catriona bought her sunglasses, bag and watch for 40%, 30% and 50% of the marked prices respectively. She spent a total of 35 euro.

(ii) Show that x, y and z also satisfy the equation

$$4x + 3y + 5z = 350$$ [1]

Grace bought her sunglasses, bag and watch for 50%, 50% and 30% of the marked prices respectively. She spent a total of 39 euro.

(iii) Show that x, y and z also satisfy the equation

$$5x + 5y + 3z = 390$$ [1]

(iv) Solve these equations, showing clearly each stage of your solution. [8]

Another friend, Laura, had asked Grace to buy her a pair of sunglasses and a watch. Grace bought these at the same prices as she herself had paid for them.

(v) Would Laura have spent less money if she had asked one of the other girls to buy the sunglasses and watch at the same prices as they had paid for them? [2]

11 A curve is defined by the equation

$$y = x^3 - 8x^2 + 16x$$

(i) Find the coordinates of the two points where the curve meets the x-axis. [2]

(ii) Find the coordinates of the turning points and identify each as either a maximum or a minimum point. [8]

(iii) **Sketch** the curve using **Fig. 6** in your Supplementary Answer Booklet. [3]

(iv) Find the area enclosed by the curve and the x-axis. [3]

THIS IS THE END OF THE QUESTION PAPER

Centre Number

71	

Candidate Number

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TUESDAY 13 MAY, AFTERNOON

G0301SAB

SUPPLEMENTARY ANSWER BOOKLET

1 **(i)** **Sketch** the graph of $y = \sin x$ on the axes in **Fig. 1** below.

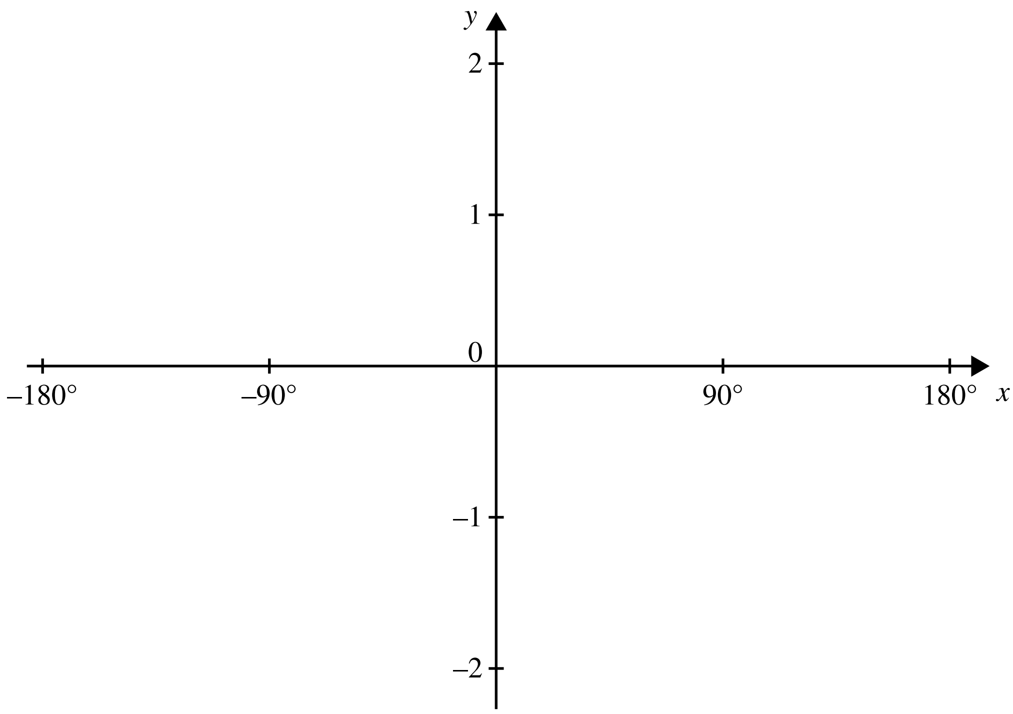

Fig. 1

(ii) **Sketch** the graph of $y = \sin 2x$ on the axes in **Fig. 2** below.

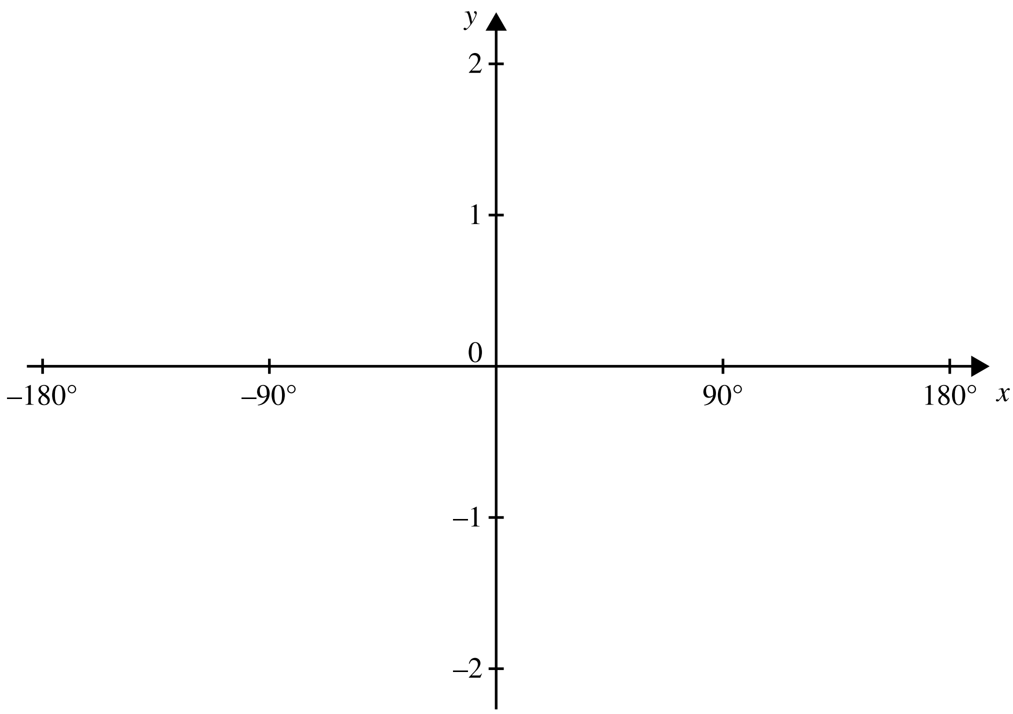

Fig. 2

9 Draw a suitable straight line graph using the axes and scales in **Fig. 5** below.
Label the axes.

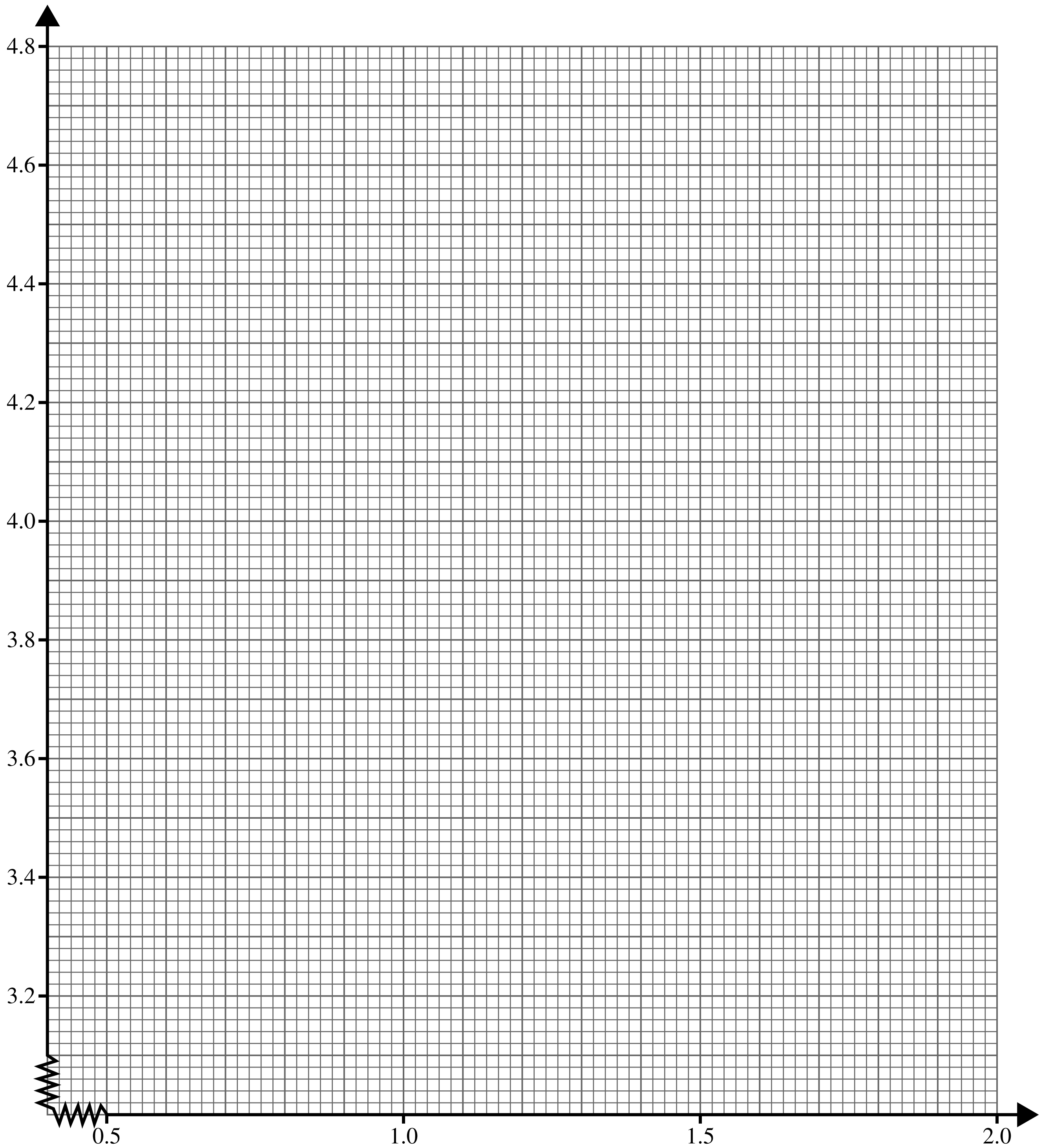

Fig. 5

11 **Sketch** the graph of $y = x^3 - 8x^2 + 16x$ in **Fig. 6** below.

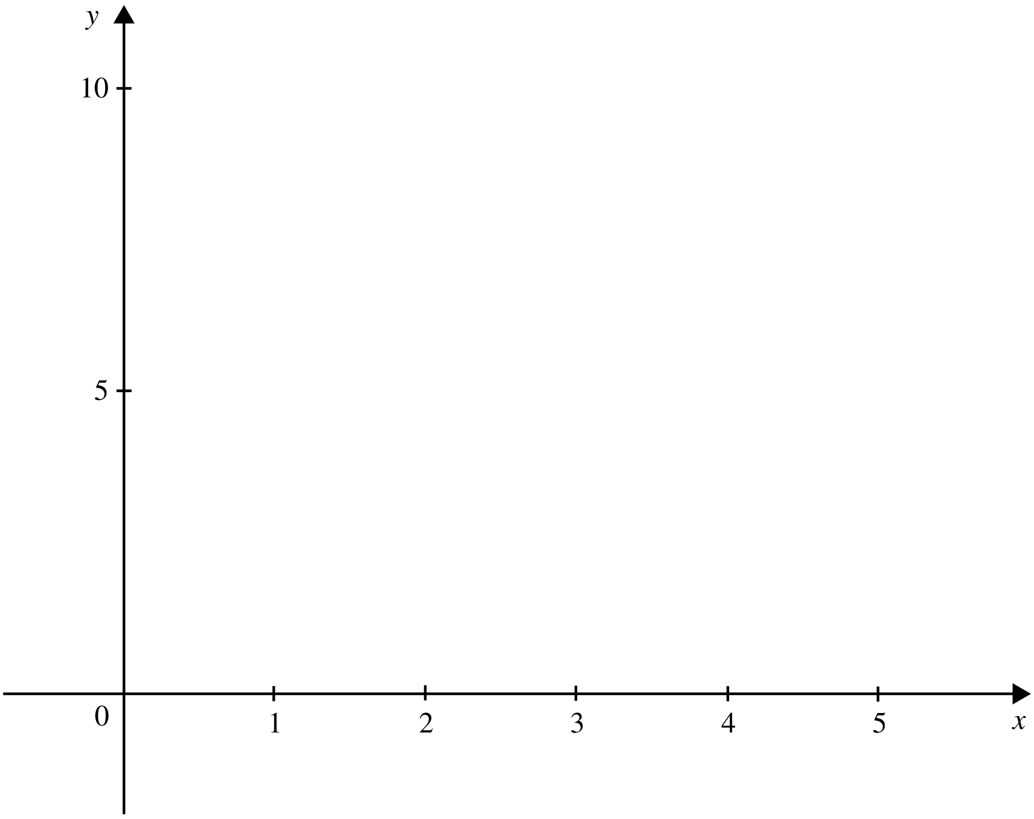

Fig. 6